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Analog Systems & Applications

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5. Coupled Amplifier

6. Sinusoidal Oscillators

Chapter 5
Coupled amplifier

Coupled Amplifier: Two stage RC-coupled amplifier and its frequency response.

(4 Lectures)

Q: Explain RC coupled amplifier using a diagram. Derive the expression for voltage gain in low frequency, mid frequency and high frequency region.

Ans:

The cascaded amplifier where an *RC* network is used for interstage coupling is known as a resistance-capacitance (*RC*) coupled amplifier. A two-stage *RC* coupled transistor amplifier in the CE configuration is depicted in Fig. 9.2. The *coupling capacitor C* couples the output signal of the first stage to the input of the second stage. The capacitor blocks the dc voltage at the output of the first stage from appearing at the input of the second stage, but it allows the ac components of the output signal to pass through it. The quiescent operating point is determined by the supply voltage V_{CC} together with the resistances R_1, R_2, R_L and R_E. The *bypass capacitor* C_E in shunt with R_E has a very small reactance at the lowest signal frequency.

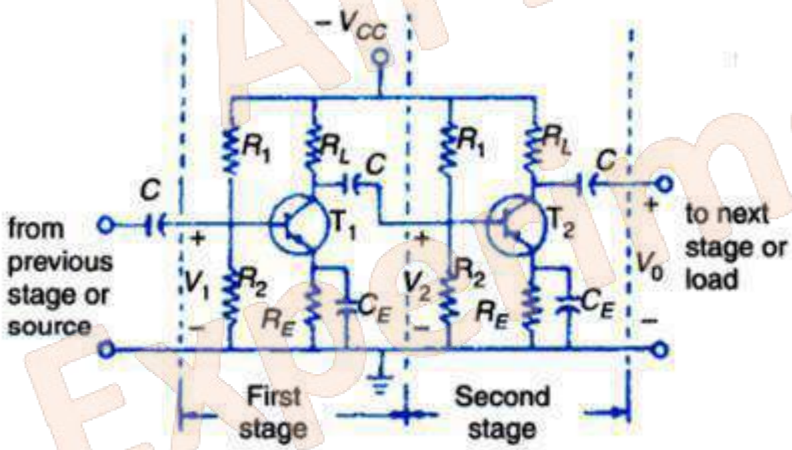

Fig. 9.2 A two-stage *RC*-coupled CE transistor amplifier.

The voltage gain of one stage, say the first stage, of the *RC*-coupled amplifier is

$$A_V = \frac{V_2}{V_1} = |A_V| \angle\theta. \tag{9.2}$$

The variation of the magnitude and the phase angle of the gain of an amplifier with frequency is referred to as the *frequency response characteristic* of the amplifier. A plot of the magnitude of the voltage gain $|A_V|$ with frequency for one stage of an *RC*-coupled amplifier is shown in Fig. 9.3. The plot of the phase angle θ of the voltage gain of the stage versus frequency is shown in Fig. 9.4. The frequency response characteristic of the stage has three regions: (*i*) the *mid-frequency range* where the voltage gain $|A_V|$ is approximately constant and the phase angle θ is 180° over a range of frequencies, (*ii*) the *low-frequency range* where the gain $|A_V|$ decreases and the phase angle increases over 180° with decreasing frequency below the mid-frequency range, and (*iii*) the *high-frequency range* where the gain $|A_V|$ falls off and the phase angle θ decreases below 180° with increasing frequency above the mid-frequency range.

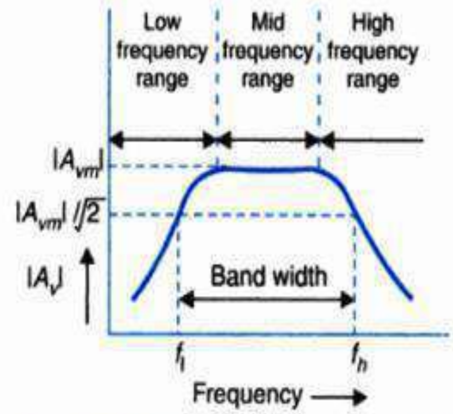

Fig. 9.3 $|A_v|$ versus frequency plot for one stage of an *RC*-coupled amplifier.

The fall of $|A_V|$ and the increase of θ over 180° with decreasing frequency in the low-frequency range are accounted for by the coupling capacitor C. As the reactance of a capacitor increases with diminishing frequency, the voltage drop across C becomes more important at low frequencies. The decrease of $|A_V|$ and the fall of θ below 180° in the high-frequency range are primarily determined by the transistor collector capacitance and the wiring capacitances appearing in shunt across the output. The drop of h_{fe} at high frequencies also contributes to the high-frequency response.

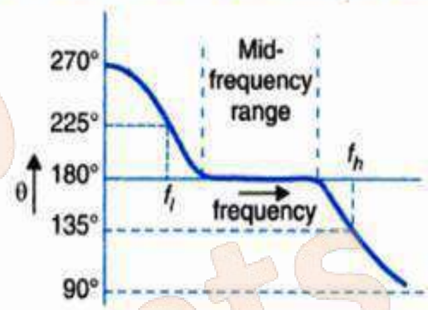

Fig. 9.4 θ versus frequency plot for one stage of an *RC*-coupled amplifier.

A. Mid-frequency Gain

In the mid-frequency range, the coupling capacitor C has a negligible reactance. Also, the output collector capacitor and other stray capacitors in shunt with the output are taken to be open-circuited. Replacing the transistor by its approximate hybrid model (see Sec. 8.9), we obtain the equivalent circuit of Fig. 9.5 for the first stage of the *RC*-coupled amplifier of Fig. 9.2.

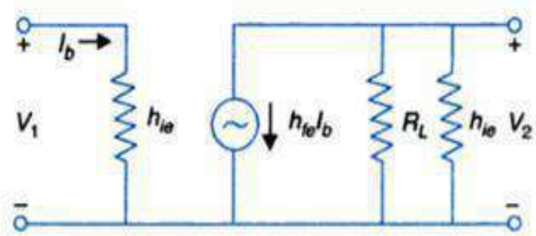

Fig. 9.5 Mid-frequency ac equivalent circuit of one stage of *RC*-coupled CE transistor amplifier.

The parallel combination of R_L and h_{ie} at the output gives the effective load resistance. R'_L That is,

$$\frac{1}{R'_L} = \frac{1}{R_L} + \frac{1}{h_{ie}} \quad \text{or} \quad R'_L = \frac{h_{ie}R_L}{h_{ie} + R_L} \tag{9.3}$$

The output voltage is

$$V_2 = -h_{fe} I_b R'_L \tag{9.4}$$

The input voltage is

$$V_1 = h_{ie} I_b \tag{9.5}$$

So, the mid-frequency voltage gain is

$$A_{Vm} = \frac{V_2}{V_1} = -\frac{h_{fe} R'_L}{h_{ie}} = -\frac{h_{fe} R_L}{h_{ie} + R_L} = -\frac{h_{fe}}{1 + (h_{ie}/R_L)} \tag{9.6}$$

The negative sign in Eq. (9.6) implies that the phase angle of the voltage gain is 180°. In other words, the output voltage leads the input voltage by 180°. Note that $|A_{Vm}|$ is independent of frequency and rises with R_L, approaching h_{fe} as $R_L \to \infty$.

B. Low-frequency Gain

In the low-frequency range, the reactance of the coupling capacitor C must be included. The shunt capacitor can, however, be considered to be an open-circuit. Thus the ac equivalent circuit of one stage of the RC-coupled amplifier below the mid-frequency range is as depicted in Fig. 9.6. Here the effective load impedance Z_L consists of the series combination of h_{ie} and C, shunted by R_L.

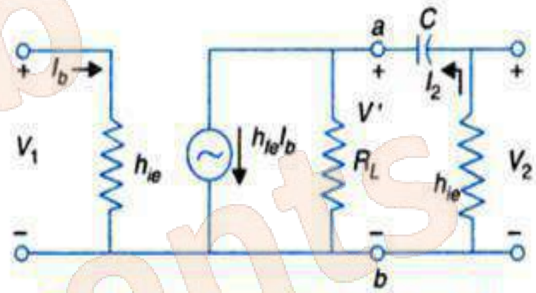

Fig. 9.6 Low-frequency ac equivalent circuit of one stage of RC-coupled CE transistor amplifier.

So,
$$\frac{1}{Z_L} = \frac{1}{R_L} + \frac{1}{h_{ie} - j/(\omega C)},$$

where ω (= $2\pi f$) is the angular frequency corresponding to the frequency f in the low-frequency range. Thus,

$$Z_L = \frac{h_{ie} - j/(\omega C)}{h_{ie} + R_L - j/(\omega C)} R_L \tag{9.7}$$

The voltage difference between the points a and b in Fig. 9.6 is

$$V' = -h_{fe} I_b Z_L. \tag{9.8}$$

If I_2 is the current through h_{ie} and C in series, we have

$$I_2 = -\frac{V'}{h_{ie} - j/(\omega C)} = \frac{h_{fe} I_b Z_L}{h_{ie} - j/(\omega C)}$$

The output voltage is

$$V_2 = -h_{ie} I_2 = -\frac{h_{ie} h_{fe} I_b Z_L}{h_{ie} - j/(\omega C)} \tag{9.9}$$

The input voltage is

$$V_1 = h_{ie} I_b. \tag{9.10}$$

From Eqs. (9.9) and (9.10), we have for the low-frequency voltage gain

$$A_{Vl} = \frac{V_2}{V_1} = -\frac{h_{fe} Z_L}{h_{ie} - j/(\omega C)} = -\frac{h_{fe} R_L}{h_{ie} + R_L - j/(\omega C)}, \tag{9.11}$$

where Eq. (9.7) is used. The magnitude and the phase angle of A_{Vl} are given by

$$|A_{Vl}| = \frac{h_{fe} R_L}{\sqrt{(h_{ie} + R_L)^2 + 1/(\omega^2 C^2)}} \tag{9.12}$$

$$\theta_l = 180^\circ + \tan^{-1} \frac{1}{\omega C(h_{ie} + R_L)} \tag{9.13}$$

From Eqs. (9.11) and (9.6), we obtain

$$A_{Vl} = \frac{A_{Vm}}{1 - j/[2\pi f C(h_{ie} + R_L)]} \tag{9.14}$$

Equation (9.14) relates the low-frequency voltage gain A_{Vl} to the mid-frequency voltage gain A_{Vm}. At the lower half-power frequency f_l, $|A_{Vl}| = |A_{Vm}|/\sqrt{2}$. So, Eq. (9.14) yields at $f = f_l$

$$\frac{|A_{Vm}|}{\sqrt{2}} = \frac{|A_{Vm}|}{\sqrt{1 + 1/[2\pi f_1 C(h_{ie} + R_L)]^2}}$$

or,

$$f_l = \frac{1}{2\pi C(h_{ie} + R_L)} \tag{9.15}$$

The low-frequency response improves, i.e. f_l is lowered when C and R_L are enhanced for a given transistor. Using Eq. (9.15), Eq. (9.14) can be written as

$$A_{Vl} = \frac{A_{Vm}}{1 - j(f_l/f)}, \tag{9.16}$$

so that

$$|A_{Vl}| = \frac{|A_{Vm}|}{\sqrt{1 + (f_l/f)^2}} \tag{9.17}$$

The phase angle by which A_{Vl} leads A_{Vm} is

$$\alpha_l = \tan^{-1}(f_l/f). \tag{9.18}$$

So the phase angle θ_l by which the output voltage V_2 leads the input voltage V_1 is

$$\theta_l = 180^\circ + \tan^{-1}(f_l/f). \tag{9.19}$$

Equations (9.17) and (9.19) predict that $|A_{Vl}|$ drops and θ_l rises as f decreases. This behaviour is exhibited in Figs. 9.3 and 9.4. At $f = f_l$, we have from Eq. (9.19),

$$\theta_l = (180^\circ + 45^\circ) = 225^\circ.$$

Chapter 6
Sinusoidal oscillators

Sinusoidal Oscillators: Barkhausen's Criterion for self-sustained oscillations. RC Phase shift oscillator, determination of Frequency.Hartley & Colpitts oscillators. **(4 Lectures)**

Q: Give barkhausen's criterion for self-sustained oscillations.

Ans:

the overall gain of a feedback amplifier is $A_f = A/(1 + A\beta)$,
where A is the gain of the internal amplifier, β is the feedback ratio, and $-A\beta$ is the loop gain. If the feedback signal aids the externally applied input signal, it is convenient to write the overall gain as

$$A_f = \frac{A}{1 - A\beta}, \qquad (11.1)$$

where $A\beta$ is the loop gain. If $A\beta = 1$, Eq. (11.1) shows that $A_f = \infty$. The amplifier then gives an output voltage without requiring any externally applied input voltage. In other words, the amplifier becomes an oscillator. To see the situation more clearly, we assume that a transient disturbance gives an output signal V_0 although no input signal is applied externally. A portion βV_0 of the output signal is fedback to the input and appears at the output as an enhanced signal $A\beta V_0$. If this signal equals V_0, i.e. $A\beta V_0 = V_0$. or $A\beta = 1$, the spurious output voltage regenerates itself or the amplifier oscillates. This condition of unity loop gain, i.e. $A\beta = 1$, is called the *Barkhausen criterion*. This condition means that $|A\beta| = 1$ and the phase angle of $A\beta$ is zero or an integral multiple of 360°. Therefore, the *basic conditions* for oscillation in a feedback amplifier are: (i) the feedback must be regenerative, and (ii) the loop gain must be unity.

Q: Explain the working of Hartley oscillator with the help of diagram. Derive the expression for frequency of oscillation and condition for sustained oscillation.

Ans:

Fig. 11.6 shows the circuit of a phase-shift oscillator. The dc operating point of the transistor in the active region of its characteristics is established by the resistors R_1, R_2, R_L and R_E, and the supply voltage $-V_{cc}$. The capacitor C_E is a bypass capacitor.

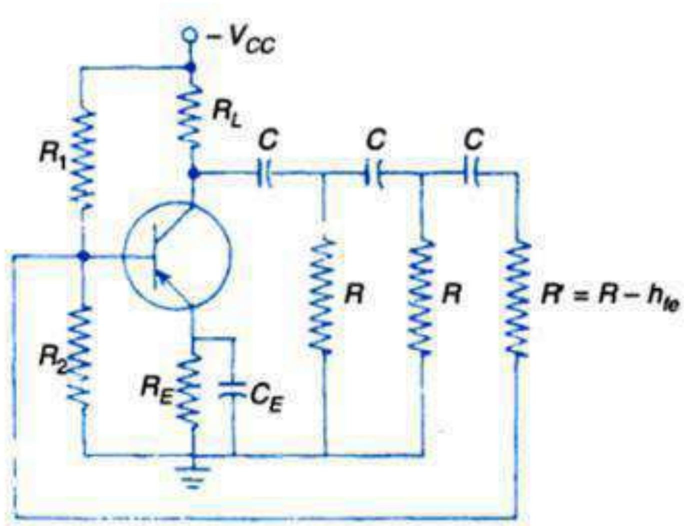

Fig. 11.6 Circuit of a phase-shift oscillator.

As the transistor is in the CE configu-ration, it introduces a phase difference of 180° between its input and output voltages. The three sections of RC network give an extra phase difference of 180° so that the net phase shift around the loop is 0° or 360°.

For convenience, the three RC sections are taken to be identical. The resistance in the last section is $R' = R - h_{ie}$. The input resistance h_{ie} of the transistor is added to R', thus giving a net resistance R. The RC phase-shift networks constitute the frequency determining circuit.

Analysis

The resistances R_1 and R_2 are large and therefore have no effect on the AC operation of the circuit. The $R_E - C_E$ parallel combination is also absent from the AC equivalent circuit of Fig. 11.7(a) due to the negligible impedance offered by this combination to ac. As $1/h_{oe} >> R_L$, one can neglect $1/h_{oe}$ which is in parallel with R_L. Also, h_{re} being small, $h_{re}\,V_2$ can be omitted. The equivalent circuit of Fig. 11.7(a) then simplified to the circuit of Fig. 11.7(b) where the current source is replaced by its equivalent voltage source.

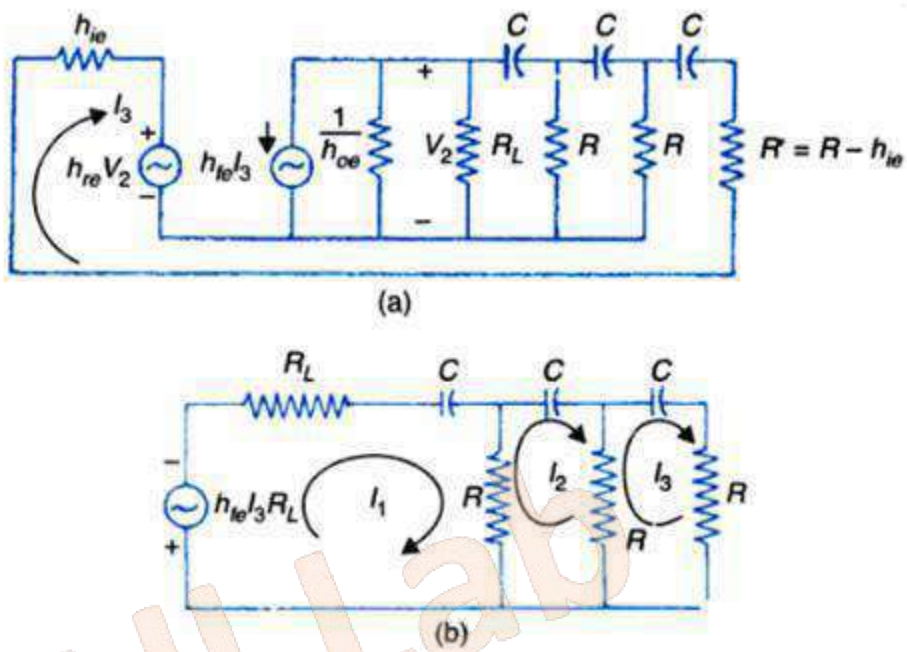

Fig. 11.7 (a) AC equivalent circuit of Fig. 11.6, (b) Simplified AC equivalent circuit with h_{re} V_2 and $1/h_{oe}$ neglected.

Kirchhoff's voltage law equations for the three loops in Fig. 11.7(*b*) are respectively written as

$$\left(R_L + R - \frac{j}{\omega C}\right) I_1 - RI_2 + h_{fe} R_L I_3 = 0 \tag{11.35}$$

$$-RI_1 + \left(2R - \frac{j}{\omega C}\right) I_2 - RI_3 = 0 \tag{11.36}$$

and

$$0.I_1 - RI_2 + \left(2R - \frac{j}{\omega C}\right) I_3 = 0 \tag{11.37}$$

where ω is the angular frequency of oscillation. Since the currents I_1, I_2, and I_3 are nonzero, the determinant of the coefficients of I_1, I_2, and I_3 in Eqs. (11.35) through (11.37) must vanish. Thus

$$\begin{vmatrix} R_L + R - \frac{j}{\omega C} & -R & h_{fe} R_L \\ -R & 2R - \frac{j}{\omega C} & -R \\ 0 & -R & 2R - \frac{j}{\omega C} \end{vmatrix} = 0. \tag{11.38}$$

$$\left(R_L + R - \frac{j}{\omega C}\right)\left(3R^2 - j\frac{4R}{\omega C} - \frac{1}{\omega^2 C^2}\right)$$

$$-R^2\left(2R-\frac{j}{\omega C}\right)+h_{fe}R_LR^2=0 \tag{11.39}$$

Equating the imaginary part of Eq. (11.39) to zero, we get

$$-4R\frac{(R+R_L)}{\omega C}-\frac{1}{\omega C}\left(3R^2-\frac{1}{\omega^2C^2}\right)+\frac{R^2}{\omega C}=0$$

$$\omega^2=\frac{1}{C^2(4RR_L+6R^2)} \tag{11.40}$$

Therefore, the frequency of oscillation is

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi C(4RR_L+6R^2)^{1/2}} \tag{11.41}$$

Putting the real part of Eq. (11.39) to zero, we obtain

$$(R+R_L)\left(3R^2-\frac{1}{\omega^2C^2}\right)-\frac{4R}{\omega^2C^2}-2R^3+h_{fe}R_LR^2=0 \tag{11.42}$$

Substituting the value of ω^2 from Eq. (11.40) into Eq. (11.42) gives

$$h_{fe}=23+29\frac{R}{R_L}+4\frac{R_L}{R} \tag{11.43}$$

The *condition for sustained oscillation* is given by Eq. (11.43).
Ordinarily, R_L is taken equal to R. Then Eqs. (11.41) and (11.43) give

$$f=\frac{1}{2\sqrt{10}\,\pi CR}, \tag{11.44}$$

$$h_{fe}=56. \tag{11.45}$$

Hence, for sustained oscillations, the transistor should have an h_{fe} of 56 when $R_L=R$.

Phase-shift oscillators are commonly employed in the AF range. The frequency of oscillation here can be changed by using a ganged variable capacitor with three sections. To vary the frequency, the capacitances of the three sections are varied simultaneously. As conventional variable capacitors have capacitances in the range 50 pF to 500 pF, the frequency of oscillation can be altered in the ratio 10 : 1. For a greater variation of frequency, different resistors of resistances differing by a factor of 10 are employed.

Q: Explain the working of Hartley oscillator with the help of diagram. Derive the expression for frequency of oscillation and condition for sustained oscillation.

Ans:

Fig. 11.3 shows the circuit diagram of a Hartley oscillator. The dc operating point in the active region of the characteristics is established by the resistors R_1, R_2 and R_E and the collector supply voltage $-V_{cc}$. The capacitor C_B is the blocking capacitor, while C_E is the bypass capacitor. Since the transistor is in the CE configuration, it produces a phase shift of 180° between its input and output voltages. The output voltage appears across the tank circuit connected to the collector. The feedback voltage is a part of the output voltage, namely, V_1 appearing across the inductance L_1. The phase shift between the feedback voltage and the output voltage is 180°. Therefore, the total phase shift around the loop is 0° or 360°. The frequency-determining circuit is constituted by the capacitor C, and the inductors L_1 and L_2.

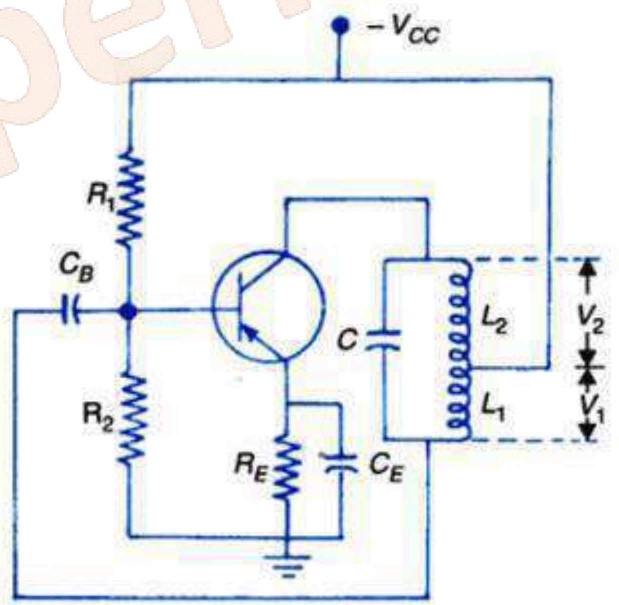

Fig. 11.3 A Hartley oscillator circuit.

Analysis

The ac operation of the circuit is not affected by the resistances R_1 and R_2 which are large. The resistance R_E is also ineffective in the ac behaviour due to the bypass capacitor C_E. The hybrid model ac equivalent circuit of the Hartley oscillator is depicted in Fig. 11.4(*a*).

We apply Thevenin's theorem looking towards the left at the terminals *X*, *Y* in the circuit of Fig. 11.4(*a*). The current source $h_{fe} I_1$ in shunt with the resistance $(1/h_{oe})$ is replaced by an equivalent Thevenin voltage source of generated voltage $(h_f I_1/h_{oe})$ and internal resistance $(1/h_{oe})$. The equivalent circuit with this Thevenin representation is given in Fig 11.4(*b*). We neglect here, for simplicity, the mutual inductance between L_1 and L_2.

From Fig. 11.4(*b*), we obtain for the voltage across the terminals *X*, *Y*

$$V_2 = \frac{1}{h_{oe}} I_2 - \frac{h_{fe}}{h_{oe}} I_1 \tag{11.13}$$

The Kirchhoff voltage law equations for the loops (1), (2), and (3) in Fig. 11.4(*b*) are, respectively,

$$\left(h_{ie} + j\omega L_1 - \frac{h_{fe}\, h_{re}}{h_{oe}}\right) I_1 + \frac{h_{re}}{h_{oe}} I_2 - j\omega L_1 I_3 = 0, \tag{11.14}$$

$$-\frac{h_{fe}}{h_{oe}} I_1 + \left(\frac{1}{h_{oe}} + j\omega L_2\right) I_2 + j\omega L_2 I_3 = 0, \tag{11.15}$$

$$-j\omega L_1 I_1 + j\omega L_2 I_2 + [j\omega L_1 + j\omega L_2 - j/(\omega C)] I_3 = 0, \tag{11.16}$$

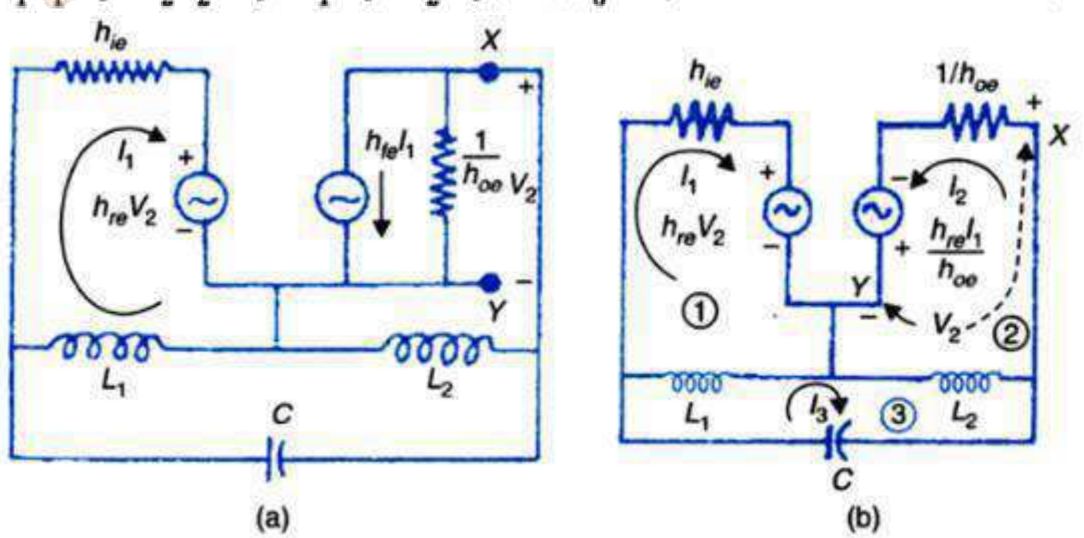

Fig. 11.4 (a) AC equivalent circuit of the Hartley oscillator. (b) AC equivalent circuit with the current source replaced by an equivalent voltage source.

where ω is the angular frequency of oscillation. Since the currents I_1, I_2 and I_3 are nonvanishing, the determinant of the coefficients of I_1, I_2 and I_3 in Eqs. (11.14) through (11.16) must be zero. That is,

$$\begin{vmatrix} h_{ie} + j\omega L_1 - \dfrac{h_{fe}\,h_{re}}{h_{oe}} & \dfrac{h_{re}}{h_{oe}} & -j\omega L_1 \\ -\dfrac{h_{fe}}{h_{oe}} & \dfrac{1}{h_{oe}} + j\omega L_2 & j\omega L_2 \\ -j\omega L_1 & j\omega L_2 & j\omega L_1 + j\omega L_2 - \dfrac{j}{\omega C} \end{vmatrix} = 0 \qquad (11.17)$$

At the frequency of oscillation, we approximately have

$$\omega(L_1 + L_2) - \frac{1}{\omega C} \approx 0 \qquad (11.18)$$

Therefore, Eq. (11.17) yields

$$\left(h_{ie} + j\omega L_1 - \frac{h_{fe}\,h_{re}}{h_{oe}}\right)L_2^2 + \frac{h_{re}}{h_{oe}}L_1L_2 - \frac{h_{fe}}{h_{oe}}L_1L_2 + \left(\frac{1}{h_{oe}} + j\omega L_2\right)L_1^2 = 0 \qquad (11.19)$$

Equating the real part of Eq. (11.19) to zero, we obtain

$$\Delta_{he}\,L_2^2 - (h_{fe} - h_{re})L_1L_2 + L_1^2 = 0 \qquad (11.20)$$

where $\Delta_{he} = h_{ie}\,h_{oe} - h_{fe}\,h_{re}$.

As $h_{re} << 1$, Eq. (11.20) reduces to

$$\Delta_{he}\,L_2^2 - h_{fe}\,L_1L_2 + L_1^2 = 0 \qquad (11.21)$$

Solving Eq. (11.21) for L_2, we get

$$L_2 = \frac{h_{fe}\,L_1 \pm \sqrt{h_{fe}^2\,L_1^2 - 4\Delta_{he}\,L_1^2}}{2\Delta_{he}} \qquad (11.22)$$

Again, $h^2_{fe} >> 4\ \Delta_{he}$. Therefore, Eq. (11.22) further simplifies to

$$L_2 \approx \frac{h_{fe}\,L_1}{\Delta_{he}} \qquad (11.23)$$

The *condition for sustaining the oscillations* is given by Eq. (11.23). Putting the imaginary part of Eq. (11.17) to zero, we obtain

$$\frac{\omega}{C} L_1 L_2 + \frac{h_{ie}}{h_{oe}} \left(\omega L_1 + \omega L_2 - \frac{1}{\omega C} \right) = 0$$

$$\omega^2 = \frac{1}{(h_{oe} \; L_1 L_2 \;/\; h_{ie}) + C(L_1 + L_2)} \qquad (11.24)$$

The *frequency of oscillation* is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \frac{1}{[(h_{oe} \; L_1 L_2 \;/\; h_{ie}) + C(L_1 + L_2)]^{1/2}} \qquad (11.25)$$

In practice, $h_{oe} L_1 L_2 / h_{ie} << C (L_1 + L_2)$, so that Eq. (11.25) gives

$$f \approx \frac{1}{2\pi \sqrt{C(L_1 + L_2)}} = \frac{1}{2\,\pi \sqrt{LC}}, \qquad (11.26)$$

where L (= $L_1 + L_2$) is the total inductance of the tank-circuit coil. *Thus the circuit gives oscillations at nearly the resonant frequency of the tank circuit.*

As mentioned earlier, we have neglected in the above analysis the mutual inductance M between L_1 and L_2. It can be shown that, when M is incorporated, L_1 and L_2 in Eqs. (11.23) and (11.26) are respectively replaced by $(L_1 + M)$ and $(L_2 + M)$.

Hartley oscillators usually generate oscillations in the *RF* range since the required values of L and C are convenient from practical considerations. The frequency of oscillation can be altered by varying L or C or both. The capacitance C can be easily changed, but smooth variations of L over a large range are inconvenient. So, for a smooth variation of frequency, capacitive tuning is preferred. Since variable capacitors have capacitances in the range 50 pF to 500 pF, Eq. (11.26) shows that the frequency of oscillation can be changed in the ratio 3 : 1.

Q: Explain the working of colpitts oscillator with the help of diagram. Derive the expression for frequency of oscillation and condition for sustained oscillation.

Ans:

A Colpitts oscillator circuit is shown in Fig. 11.5(*a*). The dc operating point of the transistor in the active region of its characteristics is established by the resistors R_1, R_2, R_L, and R_E, and the supply voltage $-V_{cc}$. The capacitor C_B blocks the dc current flow from the collector to the base of the transistor through the coil of inductance L. The capacitor C_E is a bypass capacitor. The reactances of C_E and C_B are negligible at the frequency of oscillation. The inductance L and the capacitances C_1 and C_2 constitute the frequency-determining network.

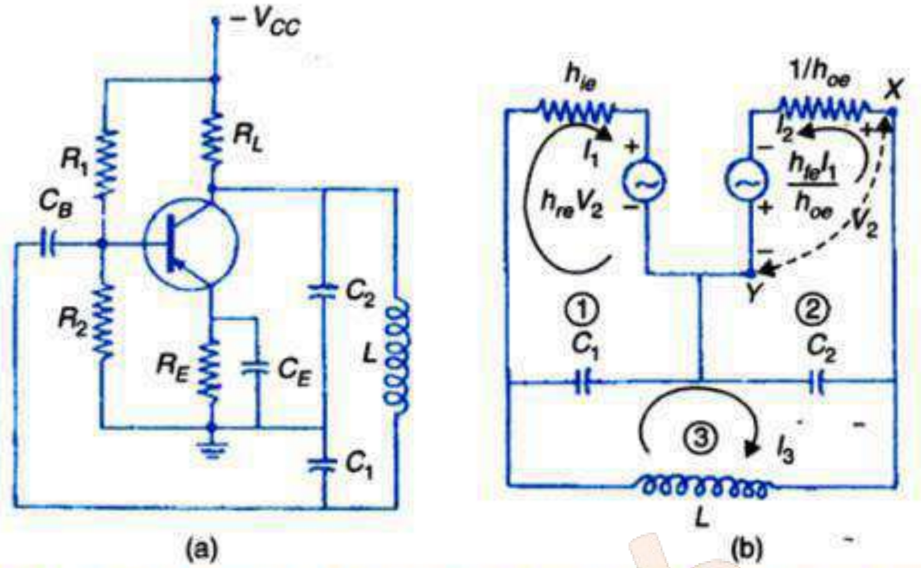

Fig. 11.5 (a) The circuit of a Colpitts oscillator, (b) Its AC equivalent circuit.

The transistor being in the CE configuration, introduces a phase shift of 180° between its input and output voltages. The voltage across the capacitor C_1, which is a fraction of the output voltage, is the feedback voltage. As the feedback voltage is 180° out of phase with the output voltage, the phase shift around the loop is 0° or 360°. It is noticed that Hartley and Colpitts oscillators are similar with the inductance and the capacitance interchanged.

Since R_1 and R_2 are large resistances, they do not affect the ac operation of the circuit. Also, R_E, being shunted by C_E which bypasses the ac, is excluded from the AC equivalent circuit, shown in Fig. 11.5 (*b*). Also, R_L is omitted since it is much larger than $1/h_{oe}$, and the current source is transformed into a voltage source in the AC equivalent circuit to facilitate the analysis.

The potential difference between the points *X*, *Y* in Fig. 11.5(*b*) is

$$V_2 = \frac{1}{h_{oe}} I_2 - \frac{h_{fe}}{h_{oe}} I_1 \tag{11.27}$$

Applying Kirchhoff's voltage law to loops (1), (2) and (3) in Fig. 11.5 (*b*), we obtain respectively

$$\left(h_{ie} - \frac{h_{fe}\,h_{re}}{h_{oe}} - \frac{j}{\omega C_1}\right) I_1 + \frac{h_{re}}{h_{oe}} I_2 + \frac{j}{\omega C_1} I_3 = 0 \tag{11.28}$$

$$-\frac{h_{fe}}{h_{oe}} I_1 + \left(\frac{1}{h_{oe}} - \frac{j}{\omega C_2}\right) I_2 - \frac{j}{\omega C_2} I_3 = 0 \tag{11.29}$$

and $$\frac{j}{\omega C_1} I_1 - \frac{j}{\omega C_2} I_2 + j\left(\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2}\right) I_3 = 0. \tag{11.30}$$

where Eq. (11.27) has been used. At the angular frequency ω of oscillation, the tuned circuit is nearly resonant so that $\omega L - \frac{1}{\omega C_1} - \frac{1}{\omega C_2} = 0$, Assuming $h_{re} << 1$, and $h^2_{fe} >> 4\Delta_{he}$, the condition for the sustained oscillations and the frequency of oscillation can be derived in the same manner as for a Hartley oscillator. Thus, equating the real part of the determinant of the coefficients of I_1, I_2 and I_3 in Eqs. (11.28) through (11.30) to zero, we get

$$\frac{C_1}{C_2} \approx \frac{h_{fe}}{\Delta_{he}}, \tag{11.31}$$

where $\Delta_{he} = h_{ie}\, h_{oe} - h_{fe}\, h_{re}$. The *condition for sustained oscillations* in the Colpitts oscillator is given by Eq. (11.31). As $h_{fe} \approx 50$ and $\Delta_{he} \approx 0.5$, Eq. (11.31) shows that $C_1/C_2 \approx 100$.

Putting the imaginary part of the determinant of the coefficients of I_1, I_2 and I_3 in Eqs. (11.28) through (11.30) to zero, we get

$$\omega^2 = \frac{h_{oe}}{h_{ie}\, C_1 C_2} + \frac{1}{LC_1} + \frac{1}{LC_2} \tag{11.32}$$

The *frequency of oscillation* is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left(\frac{h_{oe}}{h_{ie}\, C_1 C_2} + \frac{1}{LC_1} + \frac{1}{LC_2}\right)^{1/2} \tag{11.33}$$

In practice, $h_{oe} / (h_{ie}\, C_1\, C_2) << [1/(LC_1) + 1/(LC_2)]$. Hence Eq. (11.33) reduces to

$$f \approx \frac{1}{2\pi\sqrt{LC_s}} \tag{11.34}$$

where $1/C_s = 1/C_1 + 1/C_2$. Clearly, C_s is the equivalent capacitance of C_1 and C_2 in series. Equation (10.34) shows that the frequency of oscillation is approximately the resonant frequency of the tank circuit.

Like other *LC* oscillators Colpitts oscillators also produce RF signals. In principle, Colpitts oscillators can be tuned by changing the inductance or the capacitance of the tank circuit. However, a smooth variation of L is difficult; and for capacitive tuning, C_1 and C_2 must be simultaneously changed in the ratio of about 100 : 1. The task is not easy unless a special type of ganged variable capacitor is used. Therefore, Colpitts oscillators are generally employed to generate signals of fixed frequencies.