Free Study Material from

All Lab Experiments

Elements of Modern Physics

Chapter - 4, 5

4. Particles in Quantum Mechanics

5. Nuclear Physics

Chapter-4

One dimensional infinitely rigid box- energy eigenvalues and eigenfunctions, normalization; Quantum dot as example; Quantum mechanical scattering and tunnelling in one dimension-across a step potential & rectangular potential barrier. **(10 Lectures)**

Q: Obtain the energy eigenvalues and the normalized eigenfunctions for a particle in a one-dimensional infinite square well.

Ans:

Let us consider a particle of mass m confined in a region of width $2a$ from $x = -a$ to $x = +a$ by impenetrable walls. Such a system is also called a **one-dimensional box**. Inside the box the particle is free but experiences a sudden large force directed towards the origin as it reaches the points $x = \pm a$. Therefore, the potential energy for this problem is,

$$V(x) = \begin{cases} 0 & |x| < a \\ \infty & |x| > a \end{cases} \tag{7.1}$$

The potential is shown in Figure 7.1.

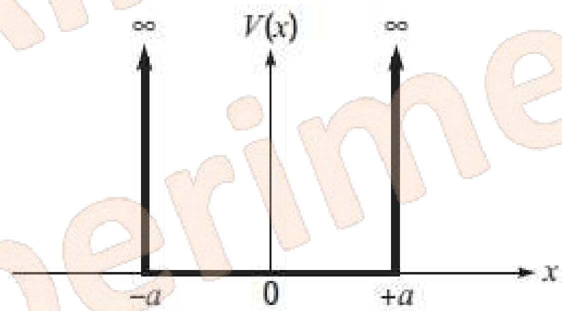


Figure 7.1 The one-dimensional infinite square well potential.

In order to find the eigenfunctions and energy eigenvalues for this system, we have to solve the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \tag{7.2}$$

Since the potential energy is infinite at $x = \pm a$, the probability of finding the particle outside the well is zero. Therefore the wave function $\psi(x)$ must vanish for $|x| > a$. Further, since the wave function must be continuous, it must vanish at the walls:

$$\psi(x) = 0 \quad \text{at} \quad x = \pm a \tag{7.3}$$

For $|x| < a$, the Schrödinger equation (7.2) reduces to

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

or

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0; \quad k^2 = \frac{2mE}{\hbar^2} \tag{7.4}$$

The general solution† of this equation is

$$\psi(x) = A \sin kx + B \cos kx \tag{7.5}$$

Applying the boundary condition (7.3), we obtain at $x = a$,

$$A \sin ka + B \cos ka = 0$$

and at $x = -a$,

$$-A \sin ka + B \cos ka = 0$$

These equations give

$$A \sin ka = 0, \quad B \cos ka = 0 \tag{7.6}$$

Now we cannot allow both A and B to be zero because this would give the physically uninteresting trivial solution $\psi(x) = 0$ for all x. Also, we cannot make both sin ka and cos ka zero for a given value of k. Hence, there are *two* possible classes of solutions:

For the *first* class,

$$A = 0 \quad \text{and} \quad \cos ka = 0$$

and for the *second* class,

$$B = 0 \quad \text{and} \quad \sin ka = 0$$

These conditions are satisfied if

$$ka = \frac{n\pi}{2} \tag{7.7}$$

where n is an *odd* integer for the first class and an *even* integer for the second class. Thus, the eigenfunctions for the two classes are, respectively,

$$\boxed{\psi_n(x) = B\cos\frac{n\pi x}{2a}, \quad n = 1,3,5,\ldots}$$

and

$$\boxed{\psi_n(x) = A\sin\frac{n\pi x}{2a}, \quad n = 2,4,6,\ldots}$$

In order to normalize the eigenfunctions, we apply the condition

$$\int_{-a}^{a} \psi_n^*(x)\ \psi_n(x)dx = 1$$

This gives

$$A^2 \int_{-a}^{a} \sin^2 \frac{n\pi x}{2a}\, dx = B^2 \int_{-a}^{a} \cos^2 \frac{n\pi x}{2a}\, dx = 1$$

Solving these integrals we obtain

$$\boxed{A = B = 1/\sqrt{a}} \tag{7.8}$$

Thus, the **normalized eigenfunctions** for the two classes are, respectively,

$$\psi_n(x) = \frac{1}{\sqrt{a}} \cos \frac{n\pi x}{2a}, \qquad n = 1,3,5,\ldots \tag{7.9}$$

and

$$\psi_n(x) = \frac{1}{\sqrt{a}} \sin \frac{n\pi x}{2a}, \qquad n = 2,4,6,\ldots \tag{7.10}$$

It may be noted that it is unnecessary to consider negative values of n because the resulting solutions will not be linearly independent of those corresponding to positive values of n.

From (7.7), the only allowed values of k are

$$k_n = \frac{n\pi}{2a}, \qquad n = 1,2,3,\ldots \tag{7.11}$$

Using (7.4) and (7.11) the **energy eigenvalues** for both the classes are given by

$$\boxed{E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{8ma^2}, \qquad n = 1,2,3,\ldots} \tag{7.12}$$

Thus, the energy is *quantized*. The integer n is called a *quantum number*. There is an *infinite* sequence of *discrete energy levels*. There is only one eigenfunction for each level, so the energy levels are *nondegenerate*.

It can be easily shown that the eigenfunctions $\psi_m(x)$ and $\psi_n(x)$ corresponding to different eigenvalues are orthogonal:

$$\int_{-a}^{a} \psi_m^*(x)\, \psi_n(x)\, dx = 0, \quad m \neq n$$

Combining orthogonality and normalization, we have the *orthonormality* condition:

$$\int_{-a}^{a} \psi_m^*(x)\,\psi_n(x)\,dx = \delta mn \tag{7.13}$$

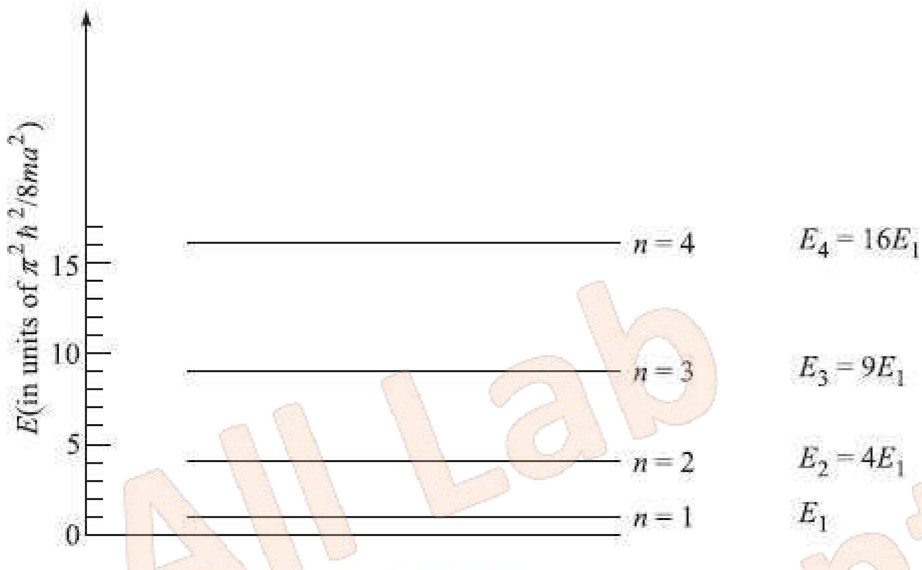


Figure 7.2 Energy-level diagram.

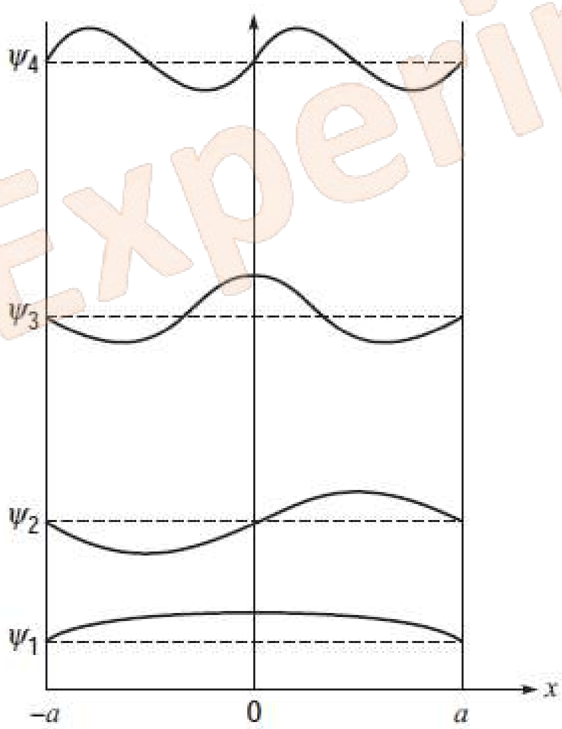


Figure 7.3 Wave functions.

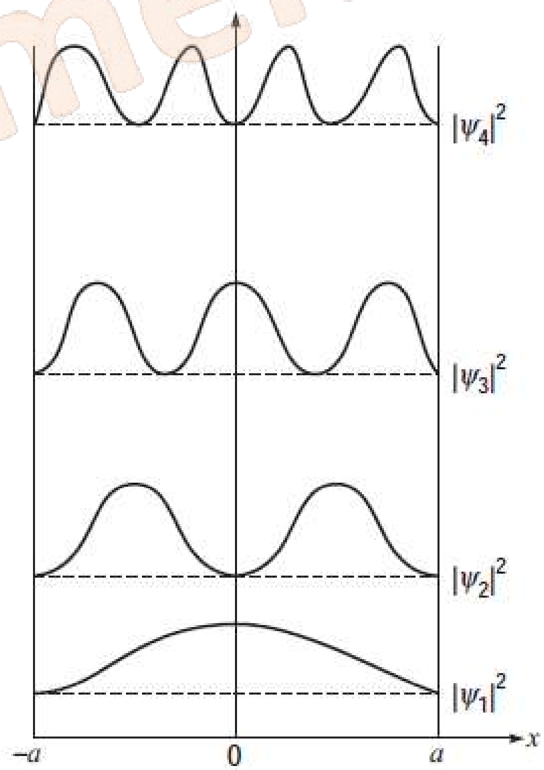


Figure 7.4 Probability densities.

Q:

Consider a particle of mass m and energy E approaching, from the left, a one-dimensional potential step given by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases}$$

Discuss the motion classically and quantum mechanically for the cases (a) $E < V_0$ and (b) $E > V_0$. Obtain the reflection and transmission coefficients.

Ans:

Let us consider the potential step shown in Figure 8.1. It is an infinite-width potential barrier given by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & x > 0 \end{cases} \tag{8.1}$$

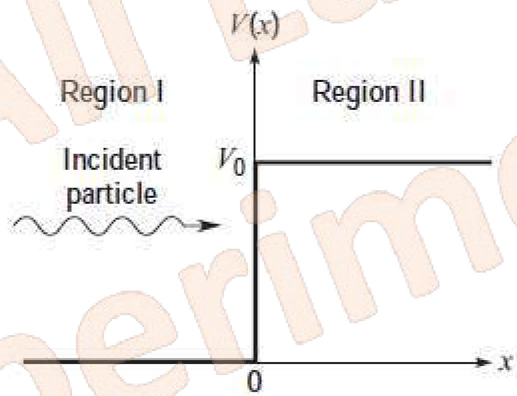


Figure 8.1 The potential step.

Suppose, a particle of mass m is incident on the step from the left with energy E. According to classical mechanics, if $E < V_0$, then the particle would be reflected back at $x = 0$ because it does not have sufficient energy to climb the barrier. On the other hand, if $E > V_0$, then the particle would not be reflected; it would keep moving towards the right with reduced energy.

We shall now study this system using quantum mechanics. Since the potential does not depend on time, the motion of the particle is described by the wave function $\Psi(x, t) = \psi(x) \exp(-iEt/\hbar)$, where $\psi(x)$ satisfies the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \qquad (8.2)$$

We shall discuss the solution of this equation separately for the two cases, $E > V_0$ and $E < V_0$.

Case 1: $E > V_0$

In **region I ($x < 0$)** the time-independent Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

or
$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0, \qquad k^2 = \frac{2mE}{\hbar^2} \qquad (8.3)$$

The general solution of this equation is

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

where A and B are arbitrary constants. We know that physically the functions exp (ikx) and exp $(-ikx)$, multiplied by the time-dependent function $\exp(-iEt/\hbar)$, represent plane waves moving towards the right and towards the left, respectively. Therefore, the first term A exp(ikx) of $\psi(x)$ in region I corresponds to a plane wave of amplitude A incident on the potential step from the left and the second term B exp$(-ikx)$ corresponds to a plane wave of amplitude B reflected from the step. Thus, according to quantum mechanics, the particle may be reflected back at $x = 0$ even though $E > V_0$. This is not possible classically.

In **region II ($x > 0$)** the Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V_0\psi = E\psi$$

or
$$\frac{d^2\psi}{dx^2} + k'^2\psi(x) = 0, \qquad k'^2 = \frac{2m(E - V_0)}{\hbar^2} \qquad (8.4)$$

Since $E > V_0$, the quantity k'^2 is positive. Therefore, the general solution of this equation is

$$\psi(x) = Ce^{ik'x} + D\,e^{-ik'x}$$

Since we are considering a particle incident on the barrier from the left, we must discard the term D exp $(-ik'x)$ which corresponds to a reflected wave in region II and there is nothing in this region which can cause such a reflection. Therefore, we must put $D = 0$. Thus the complete eigenfunction is given by

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < 0 \\ Ce^{ik'x} & x > 0 \end{cases} \tag{8.5}$$

The eigenfunction consists of an *incident* wave of amplitude A and a *reflected* wave of amplitude B in region I, and a *transmitted* wave of amplitude C in region II. The wave number in region I is k and that in region II is k'.

Continuity of ψ and $d\psi/dx$ at $x = 0$ gives

$$A + B = C$$

and

$$k(A - B) = k'C$$

from which we obtain

$$\frac{B}{A} = \frac{k - k'}{k + k'} \tag{8.6}$$

and

$$\frac{C}{A} = \frac{2k}{k + k'} \tag{8.7}$$

Let us now obtain the probability current densities associated with the incident, the reflected and the transmitted waves. From Equation (6.33) we have

$$j = \mathrm{Re}\left[\psi^* \frac{\hbar}{im}\frac{\partial\psi}{\partial x}\right]$$

If j_{in}, j_{re} and j_{tr} represent the magnitudes of the probability current densities associated with the incident, the reflected and the transmitted waves, respectively, then we readily obtain

$$j_{in} = \frac{\hbar k}{m}|A|^2 \tag{8.8}$$

$$j_{re} = \frac{\hbar k}{m}|B|^2 \tag{8.9}$$

$$j_{tr} = \frac{\hbar k'}{m}|C|^2 \tag{8.10}$$

A particle incident on the step will either be reflected or transmitted. The probability of reflection is given by the **reflection coefficient** R:

$$\boxed{R = \frac{j_{\text{re}}}{j_{\text{in}}} = \left|\frac{B}{A}\right|^2 = \left(\frac{k-k'}{k+k'}\right)^2} \qquad (8.11)$$

Substituting the values of k and k' and simplifying this becomes

$$\boxed{R = \left[\frac{1-(1-V_0/E)^{1/2}}{1+(1-V_0/E)^{1/2}}\right]^2, \quad E > V_0} \qquad (8.12)$$

The probability of transmission is given by the **transmission coefficient** T:

$$\boxed{T = \frac{j_{\text{tr}}}{j_{\text{in}}} = \frac{k'}{k}\left|\frac{C}{A}\right|^2 = \frac{4kk'}{(k+k')^2}} \qquad (8.13)$$

Substituting the values of k and k',

$$\boxed{T = \frac{4(1-V_0/E)^{1/2}}{[1+(1-V_0/E)^{1/2}]^2}, \quad E > V_0} \qquad (8.14)$$

Note that R and T depend only on the ratio V_0/E. Note also that $R + T = 1$, as it must be, because the probability is conserved.

It can be easily shown that

$$k(|A|^2 - |B|^2) = k'|C|^2 \qquad (8.15)$$

This shows that the net current incident on the step from the left is equal to the transmitted current.

Case 2: $E < V_0$

In region I the Schrödinger equation, its solution and interpretation remain the same as in case 1 ($E > V_0$). In region II the equation becomes

$$\frac{d^2\psi(x)}{dx^2} - K^2\psi(x) = 0, \quad K^2 = \frac{2m(V_0 - E)}{\hbar^2} \qquad (8.16)$$

Since $V_0 > E$, the quantity K^2 is positive. Therefore, the general solution of this equation is

$$\psi(x) = Ce^{-Kx} + De^{Kx}$$

Now, the wave function should not become infinite at $x \to \infty$. Since $\exp(Kx)$ diverges in that limit, we must choose $D = 0$. Thus the complete eigenfunction is given by

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < 0 \\ Ce^{-Kx} & x > 0 \end{cases} \tag{8.17}$$

We note that the wave function is not zero in the classically forbidden region II, although it decreases rapidly as x increases. Thus there is a finite, though small, probability of finding the particle in region II. This phenomenon is called **barrier penetration** and is observed experimentally in various atomic and nuclear systems. It illustrates a fundamental difference between classical and quantum physics.

Chapter-5

Size and structure of atomic nucleus and its relation with atomic weight; Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle. Nature of nuclear force, NZ graph, Liquid Drop model: semi-empirical mass formula and binding energy. **(6 Lectures)**

Q: what are isotopes,isobars,isotones,isomers and mirror nuclei.

Ans:

Atoms of different elements are classified as follows :

(*i*) *Isotopes* are nuclei with the same atomic number Z but different mass numbers A. The nuclei $_{14}Si^{28}$, $_{14}Si^{29}$, $_{14}Si^{30}$, and $_{14}Si^{32}$ are all isotopes of silicon. The isotopes of an element all contain the same number of protons but have different number of neutrons. Since the nuclear charge is what is ultimately responsible for the characteristic properties of an atom, all the isotopes of an element have identical chemical behaviour and differ physically only in mass.

(*ii*) Those nuclei, with the same mass number A, but different atomic number Z, are called *isobars*. The nuclei $_8O^{16}$ and $_7N^{16}$ are examples of isobars. The isobars are atoms of different elements and have different physical and chemical properties.

(*iii*) Nuclei, with an equal number of neutrons, that is, with the same N, are called *isotones*. Some isotones are $_6C^{14}$, $_7N^{15}$ and $_8O^{16}$ ($N = 8$ in each case).

(*iv*) There are atoms, which have the same Z and same A, but differ from one another in their nuclear energy states and exhibit differences in their internal structure. These nuclei are distinguished by their different life times. Such nuclei are called *isomeric nuclei* or *isomers*.

(*v*) Nuclei, having the same mass number A, but with the proton and neutron number interchanged (that is, the number of protons in one is equal to the number of neutrons in the other) are called *mirror nuclei*.

Q:Give an account of general properties of nuclei.

Ans:

Nuclear size. Rutherford's work on the scattering of α-particles showed that the mean radius of an atomic nucleus is of the order of 10^{-14} to 10^{-15} m while that of the atom is about 10^{-10} m. Thus the nucleus is about 10000 times smaller in radius than the atom.

The empirical formula for the nuclear radius is

$$R = r_o A^{1/3}$$

where A is the mass number and $r_o = 1.3 \times 10^{-15}$ m $= 1.3\ fm$. Nuclei are so small that the fermi (fm) is an appropriate unit of length. $1 fm = 10^{-15}$ m. From this formula we find that the radius of the $_6C^{12}$ nucleus is $R \approx (1.3)\ (12)^{1/3} = 3 fm$. Similarly, the radius of the $_{47}Ag^{107}$ nucleus is 6.2 fm and that of the $_{92}U^{238}$ nucleus is 8.1 fm.

Nuclear mass. We know that the nucleus consists of protons and neutrons. Then the mass of the nucleus should be

assumed nuclear mass = $Zm_p + Nm_n$.

Here, m_p and m_n are the respective proton and neutron masses and N is the neutron number. Nuclear masses are experimentally measured accurately by mass spectrometers. Measurements by mass spectrometer, however, show that

$$\text{real nuclear mass} < Zm_p + Nm_n.$$

The difference in masses

$$Zm_p + Nm_n - \text{real nuclear mass} = \Delta m$$

is called the *mass defect*.

Nuclear density. The nuclear density ρ_N can be calculated from $\rho_N = \dfrac{\text{Nuclear mass}}{\text{Nuclear volume}}$.

Nuclear mass = Am_N where A = mass number and m_N = mass of the nucleon = 1.67×10^{-27} kg.

Nuclear volume = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A$

$$\rho_N = \frac{Am_N}{\frac{4}{3}\pi r_0^3 A} = \frac{m_N}{\frac{4}{3}\pi r_0^3} = \frac{(1.67 \times 10^{-27})}{\frac{4}{3}\pi (1.3 \times 10^{-15})^3}$$

$$= 1.816 \times 10^{17} \text{ kg m}^{-3}.$$

Nuclear charge. The charge of the neucleus is due to the protons contained in it. Each proton has a positive charge of 1.6×10^{-19} C. The nuclear charge is Ze where Z is the atomic number of the nucleus. The value of Z is known from X-ray scattering experiments, from the nuclear scattering of α-particles, and from the X-ray spectrum.

Spin angular momentum. Both the proton and neutron, like the electron, have an intrinsic spin. *The spin angular momentum* is computed by $L_s = \sqrt{l(l+1)}\,\hbar$. Here the quantum number l, commonly called the spin, is equal to 1/2. The spin angular momentum, then has a value $L_s = \frac{\sqrt{3}}{2}\hbar$

Resultant angular momentum. In addition to the spin angular momentum, the protons and neutrons in the nucleus have an *orbital angular momentum*. The resultant angular momentum of the nucleus is obtained by adding the spin and orbital angular momenta of all the nucleons within the nucleus. The total angular momentum of a nucleus is given by $L_N = \sqrt{l_N(l_N+1)}\,\hbar$. This total angular momentum is called *nuclear spin*.

Nuclear magnetic dipole moments. We know that the spinning electron has an associated magnetic dipole moment of 1 Bohr magneton. *i.e.*, $\mu_e = \frac{e\hbar}{2m_e}$. Proton has a positive elementary charge and due to its spin, it should have a magnetic dipole moment. According to Dirac's theory, $\mu_N = \frac{e\hbar}{2m_p}$. Here, m_p is the proton mass.

Q: Explain binding energy.hence, calculate the binding energy of an alpha particle.

Ans:

If M is the experimentally determined mass of a nuclide having Z protons and N neutrons,

$$\text{B.E.} = \{(Zm_p + Nm_n) - M\}c^2.$$

If B.E. > 0, the nucleus is stable and energy must be supplied from outside to disrupt it into its constituents. If B.E.< 0, the nucleus is unstabe and it will disintegrate by itself.

$$\text{Mass of 2 protons + 2 neutrons} = (2 \times 1.007276 + 2 \times 1.008665)u$$
$$= 4.031882u.$$
$$\text{Mass of the } \alpha\text{-particle} = 4.001506u.$$
$$\text{Mass defect} \quad \Delta m = (4.031882 - 4.001506)\,u = 0.030376\,u$$
$$\therefore \quad \text{B.E.} = (0.030376 \times 931.3)\text{ MeV} = 28.29\text{ MeV.}$$
$$= 45.32 \times 10^{-13}\text{ J}$$

Q: Explain the stability of nucleus and binding energy curve.

Ans:

$$\text{B.E per nucleon} = \frac{\text{Total B.E. of a nucleus}}{\text{The number of nucleons it contains}}.$$

The Binding Energy per nucleon is plotted as a function of mass number A in Fig. 17.3.

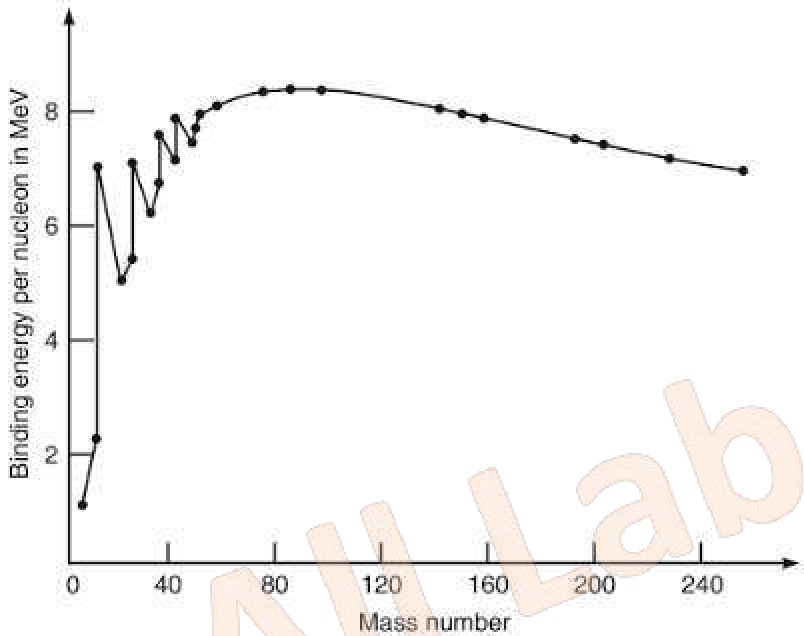


Fig. 17.3

The curve rises steeply at first and then more gradually until it reaches a maximum of 8.79 MeV at $A = 56$, corresponding to the iron nucleus $_{26}Fe^{56}$.

The curve then drops slowly to about 7.6 MeV at the highest mass numbers.

Nuclear Fission and Fusion. Evidently, nuclei of intermediate mass are the most stable, since the greatest amount of energy must be supplied to liberate each of their nucleons. This fact suggests that a large amount of energy will be liberated if heavier nuclei can somehow be split into lighter ones or if light nuclei can somehow be joined to form heavier ones. The former process is known as *nuclear fission* and the latter as *nuclear fusion*. Both the processes indeed occur under proper circumstances and do evolve energy as predicted.

Q: Give an account of stable and unstable nuclei on the basis of binding energy curve.

Ans:

Table 17.1.

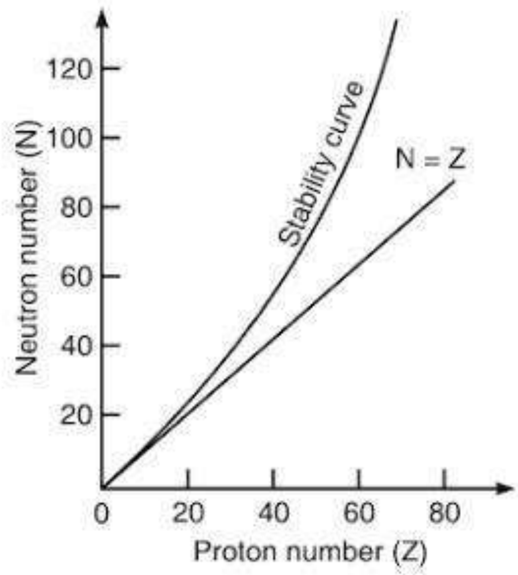
Protons	*Neutrons*	*Stable Nuclides*
even	even	160
even	odd	56
odd	even	52
odd	odd	4
		272

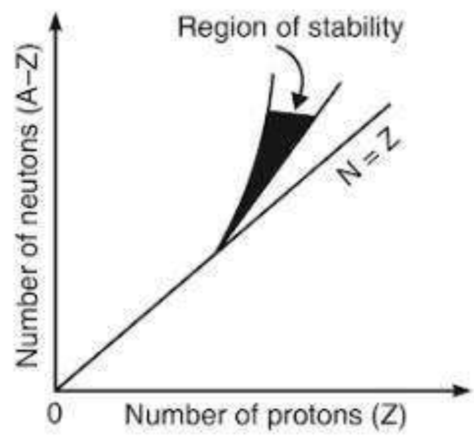
The combination of an even number of protons and an even number of neutrons, composing the nucleus, is evidently preferred by nature for stable nuclides. The odd-odd combination of stable nuclides is found only in the light elements. The number of even-odd combinations is about the same.

A plot of the number of neutrons versus the number of protons for the stable nuclides is shown in Fig. 17.5. Notice that for $Z < 20$, the stability line is a straight line with $Z = N$. For the heavier nuclides $Z > 20$, $N > 20$, the stability curve bends in the direction of $N > Z$. For example $_{20}Ca^{48}$ has $N = 28$, $Z = 20$; for larger values of Z, the tendency is more pronounced, as in the case of $_{91}Pa^{232}$ which has $N = 141$, $Z = 91$.

Evidently, for large values of Z, the coulomb electrostatic repulsion becomes important, and the number of neutrons must be greater to compensate this repulsive effect.

Thus the curve of Fig. 17.5 departs more and more from the $N = Z$ line as Z increases. For maximum stability, there is an optimum value of neutron/proton ratio. The number of neutrons N $(= A - Z)$ required for maximum stability is plotted as a function of proton number Z in Fig. 17.6. All the stable nuclei fall within the shaded region. Nuclei above and below the shaded region are unstable. Artificial radioactive nuclei lie at the fringe of the region of stability. All nuclei with $Z > 83$, and $A > 209$ spontaneously transform themselves into lighter ones through the emission of α and β particles. α and β decays enable an unstable nucleus to reach a stable configuration.





Q: why electron cannot be present inside nucleus.

Ans:

(*i*) **Nuclear size.** Typical nuclei are less than 10^{-14} m in radius. If an electron exists inside a nucleus, the uncertainty in its position (Δx) may not exceed 10^{-14} m. According to Heisenberg's uncertainty principle, uncertainty in the electron's momentum is

$$\Delta p \geq \frac{\hbar}{\Delta x} \geq \frac{1.054 \times 10^{-34}}{10^{-14}} \geq 1.1 \times 10^{-20} \text{ kg ms}^{-1}.$$

If this is the uncertainty in the electron's momentum, the momentum itself must be at least comparable in magnitude.

$\therefore$ Approximate momentum of the electron $= p = 1.1 \times 10^{-20}$ kg ms^{-1}. An electron whose momentum is 1.1×10^{-20} kg ms^{-1} has a K.E. (T) many times greater than its rest energy $m_0 c^2$ *i.e.*, $T >> m_0 c^2$. Hence we can use the extreme relativistic formula $T = pc$ to find T.

$$T = (1.1 \times 10^{-20})(3 \times 10^8) = 3.3 \times 10^{-12} J$$

$$= \frac{3.3 \times 10^{-12}}{1.6 \times 10^{-13}} MeV = 20.63 MeV.$$

This shows that if an electron exists in the nucleus, the K.E. of the electron must be more than 20 MeV. Electrons of such large energy are never found to be emitted during β-decay. The maximum energy of a β-particle emitted is only 2 to 3 MeV. Hence we conclude that electrons cannot be present within nuclei.

(*ii*) **Nuclear spin.** Electrons and protons have a spin of 1/2. Thus nuclei with an even number of protons and electrons should have integral spins, while those with an odd number of protons and electrons should have half-integral spins. Let us consider deuteron as an example. Deuteron nucleus has 3 particles (two protons and one electron). Hence the nuclear spin of deuteron should be 1/2 or 3/2. But experiment shows that the spin of the deuteron is 1. Thus the experimental result is in contradiction to the hypothesis.

(*iii*) **Magnetic moment.** Protons and electrons are endowed with magnetic properties. The magnetic moment of an electron is about one thousand times that of a proton. If electrons exist inside the nucleus, the magnetic moment of electrons will have a dominating influence and so nuclear magnetic moments ought to be of the same order of magnitude as that of the electron. However, the observed magnetic moments of nuclei are comparable with that of the proton. This experimental fact goes against the electrons existing inside the nucleus.

Due to these reasons, it is concluded that electrons cannot exist in the nucleus. Hence the proton-electron hypothesis regarding the constitution of the nucleus has been given up.

Q: Give the characteristics of nuclear forces.

Ans:

(1) Nuclear forces are effective only at short ranges. Nuclear forces are appreciable only when the distance between nucleons is of the order of 10^{-15} m or less. The force vanishes for all practical purposes at distances greater than a few times 10^{-15} m. These distances are called the *action radii* or *range* of the nuclear forces. In the up-to-date version of the exchange theory of nuclear forces, it is supposed that interaction between nucleons is accomplished by the exchange of π-mesons. The exchange version of nuclear forces explains their short range action. Let m be the rest mass of the π-meson.

Rest energy of the π-meson $= \Delta E = mc^2$. According to Heisenberg's uncertainty principle, the time required for nucleons to exchange π-mesons cannot exceed Δt, for which $\Delta E \, \Delta t \geq \hbar$. The distance that a π-meson can travel away from a nucleon in the nucleus during the time Δt, even at a velocity $\approx c$, is

$$R_0 \approx \frac{\hbar}{mc} \approx 1.2 \times 10^{-15}\ m.$$

This approximately coincides with the value of the nuclear radius and is of the order of magnitude of the nuclear force range.

(2) Nuclear forces are charge independent. The nuclear forces acting between two protons, or between two neutrons, or between a proton and a neutron, are the same. It follows that nuclear forces are of a non-electric nature.

(3) Nuclear forces are the strongest known forces in nature.

(4) Nuclear forces have saturation property. Nuclear forces are limited in range. As a result, each nucleon interacts with only a limited number of nucleons nearest to it. This effect is referred to as the *saturation* of nuclear forces.

Q: write the similarities between nucleus and liquid drop in liquid drop model.

Ans:

In the liquid-drop model, the forces acting in the nucleus are assumed to be analogical to the molecular forces in a droplet of some liquid. This model was proposed by Neils Bohr who observed that there are certain marked similarities between an atomic nucleus and a liquid drop. The similarities between the nucleus and a liquid drop are the following:

(*i*) The nucleus is supposed to be spherical in shape in the stable state, just as a liquid drop is spherical due to the symmetrical surface tension forces.

(*ii*) The force of surface tension acts on the surface of the liquid-drop. Similarly, there is a potential barrier at the surface of the nucleus.

(*iii*) The density of a liquid-drop is independent of its volume. Similarly, the density of the nucleus is independent of its volume.

(*iv*) The intermolecular forces in a liquid are short range forces. The molecules in a liquid drop interact only with their immediate neighbours. Similarly, the nuclear forces are short range forces. Nucleons in the nucleus also interact only with their immediate neighbours. This leads to the saturation in the nuclear forces and a constant binding energy per nucleon.

(*v*) The molecules evaporate from a liquid drop on raising the temperature of the liquid due to their increased energy of thermal agitation. Similarly, when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiations almost immediately.

(*vi*) When a small drop of liquid is allowed to oscillate, it breaks up into two smaller drops of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.

Q: write down semi empirical mass formula and explain its terms.

Ans:

Semi-empirical mass formula. The liquid-drop model can be used to obtain an expression for the binding energy of the nucleus. Weizacker proposed the semi-empirical nuclear binding energy formula for a nucleus of mass number A, containing Z protons and N neutrons. It is written as

$$\text{B.E.} = aA - bA^{2/3} - \frac{cZ(Z-1)}{A^{1/3}} - \frac{d(N-Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

where a, b, c, d and δ are constants.

Explanation of the terms. (1) The first term is called the *volume energy* of a nucleus ($E_v = aA$). The larger the total number of nucleons A, the more difficult it will be to remove the individual protons and neutrons from the nucleus. The B. E. is directly proportional to the total number of nucleons A.

(2) The nucleons, at the surface of the nucleus, are not completely surrounded by other nucleons. Hence energy of the nucleon on the surface is less than that in the interior. The number of surface nucleons depends upon the surface area of the nucleus. A nucleus of radius R has an area of $4\pi R^2 = 4\pi r_o^2 A^{2/3}$. Hence the *surface effect* reduces the B.E. by $E_s = b A^{2/3}$. The negative energy E_s is called the *surface energy* of a nucleus. It is most significant for the lighter nuclei, since a greater fraction of their nucleons are on the surface.

(3) The *electrostatic repulsion* between each pair of protons in a nucleus also contributes towards decreasing its B.E. The Coulomb energy E_c of a nucleus is the work that must be done to bring together Z protons from infinity into a volume equal to that of the nucleus. Hence $E_c \propto Z(Z-1)/2$ (the number of proton pairs in a nucleus containing Z protons) and E_c is inversely proportional to the nuclear radius $R = r_o A^{1/3}$. E_c is negative because it arises from a force that opposes nuclear stability.

(4) The fourth term $E_a = \dfrac{d(N-Z)^2}{A}$ originates from the *lack of symmetry* between the number of protons (Z) and the number of neutrons (N) in the nucleus. The maximum stability of a nucleus occurs when $N = Z$. Any departure from this introduces an asymmetry $N–Z$, which results in a decrease in stability. The decrease in the B.E. arising from this is called the asymmetric energy (E_a). This is also negative.

(5) The final correction term δ allows for the fact that *even-even* nuclei are more stable than *odd-odd* nuclei. δ is positive for even-even nuclei, is negative for odd-odd nuclei and $\delta = 0$ for an odd A.

The best values of the constants, expressed in MeV, are $a = 15.760$; $b = 17.810$, $c = 0.711$, $d = 23.702$, $\delta = 34$.

The contributions of the various effects in Weizacker's empirical formula are represented schematically in the graph of Fig. 17.11.

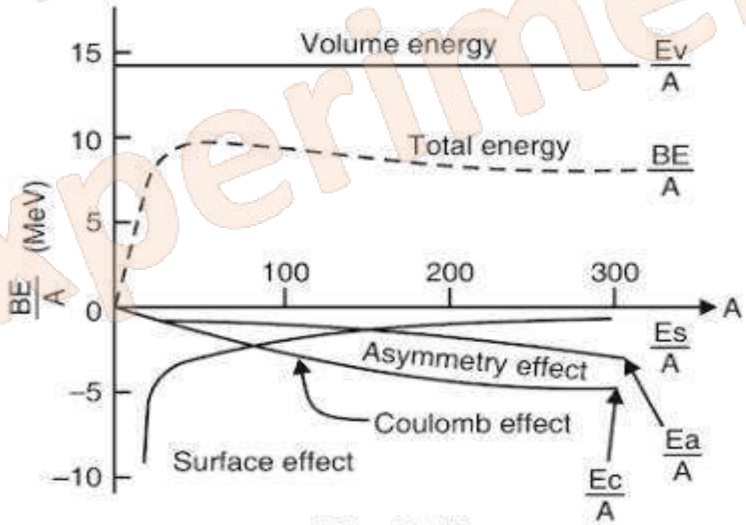


Fig. 17.11

Q: give merits and demerits of liquid drop model.

Ans:

Merits. (1) The liquid drop model accounts for many of the salient features of nuclear matter, such as the observed binding energies of nuclei and their stability against α and β disintegration as well as nuclear fission.

(2) The calculation of atomic masses and binding energies can be done with good accuracy with the liquid drop-model.

However, this model fails to explain other properties, in particular the magic numbers. It fails to explain the measured spins and magnetic moments of nuclei.